Supersymmetric branes with (almost) arbitrary tensions

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We present a supersymmetric version of the two-brane Randall-Sundrum scenario, with arbitrary brane tensions T_1 and T_2, subject to the bound $|T_{1,2}|\leqslant\sqrt{-6\Lambda_5}$, where $\Lambda_5<0$ is the bulk cosmological constant. Dimensional reduction gives $N=1$, $D=4$ supergravity, with the cosmological constant Λ_4 in the range $\frac{1}{2}\Lambda_5\leqslant\Lambda_4\leqslant 0$. The case with $\Lambda_4=0$ requires $T_1=-T_2=\sqrt{-6\Lambda_5}$. This work unifies and generalizes previous approaches to the supersymmetric Randall-Sundrum scenario. It also shows that the Randall-Sundrum fine-tuning is not a consequence of supersymmetry.

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I. INTRODUCTION

During the past few years, codimension-1 branes have been the subject of intense activity. Much of this work was sparked by Randall and Sundrum, who showed how codimension-1 branes can solve the gauge hierarchy problem [1]. In this paper we consider supersymmetric extensions of the original five-dimensional Randall-Sundrum scenario. We compactify the fifth dimension on an $S^1/\mathbb{Z}_2$ orbifold, and place codimension-1 branes at the orbifold fixed points. We require odd bosonic fields to be continuous across the branes, but we let odd fermionic fields jump in a way that is consistent with their five-dimensional equations of motion.

In previous work on this subject, the brane tensions were tuned to be equal and opposite, and equal in magnitude to the five-dimensional bulk cosmological constant, appropriately normalized [2–5]. In this paper we relax this condition and allow arbitrary brane tensions. We present a bulk-plus-brane action and find the conditions under which it is locally supersymmetric. Our results imply that the Randall-Sundrum fine-tuning is not a consequence of supersymmetry: it must be imposed by hand to obtain a flat effective four-dimensional theory.[1] More generally, our construction allows the locally supersymmetric five-dimensional theory to have effective four-dimensional theory with a negative or zero cosmological constant.

Moreover, in previous work, the bulk gravitino mass was taken to be either even [2] or odd [3–5] under the $\mathbb{Z}_2$ parity. The approach presented here allows the results to be continuously connected—even in the case of arbitrary brane tensions. Our results show that there is no conceptual difference—at least in the absence of matter—between the two cases previously considered in the literature.

The paper is organized as follows. In Sec. II we present the supersymmetric bulk-plus-brane action, together with the corresponding supersymmetry transformations. We find that supersymmetry requires the brane tensions to have magnitudes less than or equal to the bulk cosmological constant, appropriately normalized. In Sec. III we compute the low energy effective action. We keep the radion field fixed and derive the effective action for the four-dimensional supergravity multiplet. We show that the reduction leads to a locally supersymmetric theory with a negative or zero cosmological constant. We summarize our conventions and present details of our calculation in Appendixes A and B.

II. LOCALLY SUPERSYMMETRIC BULK-PLUS-BRANE SYSTEM

A. Bulk-plus-brane action

In this section we present our five dimensional bulk-plus-brane action. We start with pure $N=2$, $D=5$ supergravity, with cosmological constant $\Lambda_5=-6\lambda^2$. The cosmological constant arises from gauging a $U(1)$ subgroup of the $SU(2)$ automorphism group, determined by a unit vector of real parameters $\vec{q}=(q_1,q_2,q_3)$. The action and supersymmetry transformations [7] are given by[2]

$$S_{\rm bulk}=\int d^5x e_5\bigg\{-\frac{1}{2}R+6\lambda^2+\frac{i}{2}\tilde{\Psi}^i_M\Gamma^{MNK}D_N\Psi_{Ki}-\frac{3}{2}\lambda\vec{q}\cdot\vec{\sigma}_i{}^j\tilde{\Psi}^i_M\Sigma^{MN}\Psi_{Nj}-\frac{1}{4}F_{MN}F^{MN}$$
$$-i\frac{\sqrt{6}}{16}F_{MN}(2\tilde{\Psi}^{Mi}\Psi^N_i+\tilde{\Psi}^i_P\Gamma^{MNPQ}\Psi_{Qi})-\frac{1}{6\sqrt{6}}\epsilon^{MNPQK}F_{MN}F_{PQ}B_K+\frac{\sqrt{6}}{4}\lambda\vec{q}\cdot\vec{\sigma}_i{}^jB_N\tilde{\Psi}^i_M\Gamma^{MNK}\Psi_{Kj}\bigg\} \quad (2.1)$$

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[1]Zucker came to a similar conclusion using the off-shell formalism [6], but he was unable to find the Killing spinor that describes the unbroken supersymmetry.

[2]The supersymmetry is $N=2$ because it corresponds to a supersymmetry algebra with two independent supercharges, each belonging to the smallest spinor representation (pseudoreal, of real dimension 4) of the Lorentz group $SO(1,4)$ [8]. However, because this is the minimal algebra in $D=5$, this theory is sometimes called $N=1$.

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and

$$\delta e_M^A = i\tilde{\mathcal{H}}^i \Gamma^A \Psi_{Mi}$$

$$\delta B_M = i\frac{\sqrt{6}}{2}\tilde{\Psi}^i_M \mathcal{H}_i$$

$$\delta\Psi_{Mi} = 2\left(D_M \mathcal{H}_i - i\frac{\sqrt{6}}{2}\lambda \vec{q}\cdot\vec{\sigma}_i{}^j B_M \mathcal{H}_j \right) + i\lambda \vec{q}\cdot\vec{\sigma}_i{}^j \Gamma_M \mathcal{H}_j + \frac{1}{2\sqrt{6}}(\Gamma_{MNK} - 4g_{MK}\Gamma_N)F^{NK}\mathcal{H}_i, \tag{2.2}$$

where we drop all three- and four-Fermi terms, and the spinors are symplectic Majorana (see Appendix A). For the case at hand, we write a symplectic Majorana spinor Ψ_i as follows:

$$\Psi_1 = -\Psi^2 = \begin{pmatrix} \psi_{1\alpha} \\ \bar{\psi}_2^{\dot{\alpha}} \end{pmatrix}, \quad \Psi_2 = \Psi^1 = \begin{pmatrix} -\psi_{2\alpha} \\ \bar{\psi}_1^{\dot{\alpha}} \end{pmatrix}, \tag{2.3}$$

where $\psi_{1\alpha}$ and $\psi_{2\alpha}$ are two-component Weyl spinors.

We take the fifth dimension to span the orbifold $\mathbb{R}/\mathbb{Z}_2$. We work on the covering space and require that the action and supersymmetry transformations be invariant under reflection $z\to -z$, where $z=x^5$ is the coordinate in the fifth dimension. (We consider $S^1/\mathbb{Z}_2$ later in this section.) We assume the action is even under reflection. We then choose e_m^a, η_1 and λ to be even, which fixes the remaining parity assignments:

$$\text{even: } \partial_m,\ e_m^a,\ e_5^{\hat{5}},\ B_5,\ \eta_1,\ \psi_{m1},\ \psi_{52},\ q_{1,2},\ \lambda$$

$$\text{odd: } \partial_5,\ e_m^{\hat{5}},\ e_5^a,\ B_m,\ \eta_2,\ \psi_{m2},\ \psi_{51},\ q_3, \tag{2.4}$$

where (η_1,η_2), (ψ_{m1},ψ_{m2}) and (ψ_{51},ψ_{52}) are the two-component spinors in $\mathcal{H}_i$, Ψ_{mi} and Ψ_{5i}, respectively.

The orbifold fixed point at $z=0$ can be viewed as a 3-brane, Σ, parametrized by the coordinates $x^m=(x^0,x^1,x^2,x^3)$. We take even fields to be continuous across the brane. Odd fields, in general, can be discontinuous across Σ, with a jump that is twice their value on either side of the brane. We make the effects of this discontinuity explicit by redefining all odd fields and parameters as follows:

$$e_m^{\hat{5}} \to \varepsilon(z) e_m^{\hat{5}},\quad e_5^a \to \varepsilon(z) e_5^a,\quad B_m \to \varepsilon(z) B_m$$

$$\eta_2 \to \varepsilon(z)\eta_2,\quad \psi_{m2}\to\varepsilon(z)\psi_{m2},\quad \psi_{51}\to\varepsilon(z)\psi_{51}$$

$$q_3 \to \varepsilon(z) q_3, \tag{2.5}$$

where $\varepsilon(z)$ is the sign function on $\mathbb{R}$. After this redefinition, all fields are *even*; the odd parity arises because of the $\varepsilon(z)$ terms. In what follows, we write all expressions in terms of the redefined fields.

The discontinuities in the fields are induced by the brane action. We take

$$S_{\text{brane}} = \int_\Sigma d^4x e_4[-3\lambda_1 - 2\alpha_1 \psi_{m1}\sigma^{mn}\psi_{n1} + \text{H.c.}] = \int d^5x e_4[-3\lambda_1 - 2\alpha_1\psi_{m1}\sigma^{mn}\psi_{n1} + \text{H.c.}]\delta(z). \tag{2.6}$$

The bosonic piece describes the brane tension, $T_1 = 6\lambda_1$. The fermionic term is necessary to supersymmetrize the full bulk-plus-brane system for arbitrary $\vec{q}$.

The bulk-plus-brane equations of motion can be readily computed. In terms of the redefined variables, the bosonic equations contain terms proportional to $\delta'(z)$ and $\delta(z)^2$. We eliminate these terms by demanding that all odd bosonic fields vanish on the brane,

$$e_m^{\hat{5}} = e_5^a = B_m = 0, \tag{2.7}$$

on Σ. (This implies that e_m^a on Σ is the induced veirbein, so d^4xe_4 is the invariant integration measure on the brane.) The equations of motion for e_m^a and ψ_{m1} contain terms proportional to $\delta(z)$. These terms cancel when

$$\omega_{ma\hat{5}} = \varepsilon(z)\lambda_1 e_{ma} \tag{2.8}$$

$$\psi_{m2} = \alpha_1 \psi_{m1}, \tag{2.9}$$

on Σ. The first condition restricts the spin connection on Σ; it is a Neumann boundary condition for the metric g_{mn}. The second condition identifies the two gravitini on the brane.

B. Supersymmetry transformations

Consistency of the bulk-plus-brane theory requires closure of the supersymmetry algebra and preservation of the boundary conditions on Σ. To check the closure, we convert the transformations (2.2) to two-component notation and carry out the redefinition (2.5) (see Appendix B). It is not hard to show that the algebra closes, except for the following singular term:

$$[\delta_\xi,\delta_\eta]e_5^a = \ldots + 8i\,\eta_2\sigma^a\overline{\xi_2}\,\delta(z) + \text{H.c.} \tag{2.10}$$

We cancel this term by modifying the transformation for ψ_{52}:

$$\delta\psi_{52} = \delta\psi_{52}|_{\text{old}} - 4\,\eta_2\,\delta(z). \tag{2.11}$$

This modification makes $\delta\psi_{52}$ finite on Σ and restores the consistency of the supersymmetry algebra.

Supersymmetry also requires that the boundary conditions (2.7) be preserved:

$$\delta e_m^{\hat{5}} = \delta e_5^a = \delta B_m = 0, \tag{2.12}$$

on Σ. This imposes additional boundary conditions on the fermionic fields:

$$\eta_2 = \alpha_1\eta_1 \tag{2.13}$$

$$\psi_{51} = -\alpha_1^* \psi_{52}, \tag{2.14}$$

on Σ.

The boundary conditions (2.8), (2.9) and (2.14) must themselves be maintained under supersymmetry. Equation (2.9) requires the vanishing on Σ of

$$\alpha_1 \delta\psi_{m1} - \delta\psi_{m2} = -2\,\eta_1 \partial_m \alpha_1 + iM\sigma_m \overline{\eta}_1, \tag{2.15}$$

where

$$M = \lambda_1(1+\alpha_1\alpha_1^*\varepsilon^2) + \lambda[\alpha_1 q_{12}^* + \alpha_1^* q_{12} + (\alpha_1\alpha_1^*\varepsilon^2 - 1)q_3] \tag{2.16}$$

and $q_{12} = q_1 + iq_2$. Equation (2.15) implies that α_1 is constant on Σ. It also implies $M=0$, which gives the following relation between λ_1 and α_1:

$$\lambda_1(1+\alpha_1\alpha_1^*) + \lambda[\alpha_1 q_{12}^* + \alpha_1^* q_{12} + (\alpha_1\alpha_1^* - 1)q_3] = 0, \tag{2.17}$$

where we have used the fact that $\varepsilon^2 = 1$ (on Σ and in the bulk). The variations of Eqs. (2.14) and (2.8) give boundary conditions on $\partial_5\eta_2$ and $\partial_5\psi_{m2}$, respectively.

We now have what we need to check the invariance of the bulk-plus-brane action under supersymmetry. The total variation receives three contributions: two from the bulk and one from the brane. The first contribution comes from the redefinition $q_3 \to \varepsilon(z) q_3$ in the bulk action:

$$\delta^{(1)} S_5 = \int d^5x\, e_4 [6\lambda q_3(\alpha_1\alpha_1^*\varepsilon^2 - 1) i\psi_{m1}\sigma^m \overline{\eta}_1 + \text{H.c.}]\delta(z). \tag{2.18}$$

The second contribution arises from the modification (2.11) of the supersymmetry transformation:

$$\delta^{(2)} S_5 = \int d^5x\, e_4 \{-4\alpha_1 \eta_1 \sigma^{mn} \hat{D}_m \psi_{n1} + 4\alpha_1 \psi_{n1}\sigma^{nm}\hat{D}_m \eta_1 - \sqrt{6} i \alpha_1 e_{5}^{\hat{5}} F^{m5}\psi_{m1}\eta_1 + 6[\lambda_1\alpha_1\alpha_1^*\varepsilon^2 + \lambda(\alpha_1^* q_{12} + \alpha_1\alpha_1^*\varepsilon^2 q_3)] i\psi_{m1}\sigma^m\overline{\eta}_1 + \text{H.c.}\}\delta(z). \tag{2.19}$$

The third contribution comes from the variation of the brane action. The supersymmetry transformations are induced from the bulk,

$$\delta e_m^a = i(1+\alpha_1\alpha_1^*\varepsilon^2)\eta_1\sigma^a\overline{\psi}_{m1} + \text{H.c.}$$

$$\delta\psi_{n1} = 2\hat{D}_n\eta_1 + i[\lambda(q_{12}^* + \alpha_1^*\varepsilon^2 q_3) + \lambda_1\alpha_1^*\varepsilon^2]\sigma_n\overline{\eta}_1 - \frac{2}{\sqrt{6}} i e_{5}^{\hat{5}} F^{k5}(\sigma_{nk} + g_{nk})\eta_1. \tag{2.20}$$

The variation of the brane action is then

$$\delta S_B = \int d^5x\, e_4\{-8\alpha_1\psi_{n1}\sigma^{nm}\hat{D}_m\eta_1 + \sqrt{6} i\alpha_1 e_{5}^{\hat{5}} F^{m5}\psi_{m1}\eta_1 + 6[\lambda_1(1+2\alpha_1\alpha_1^*\varepsilon^2) + \lambda(\alpha_1 q_{12}^* + \alpha_1\alpha_1^*\varepsilon^2 q_3)] i\psi_{m1}\sigma^m\overline{\eta}_1 + \text{H.c.}\}\delta(z). \tag{2.21}$$

In these expressions, we have used the boundary conditions for the fermions and the spin connection $\omega_{ma\hat{5}}$.

The supersymmetry variation of the bulk-plus-brane action is the sum of Eqs. (2.18), (2.19) and (2.21),

$$\delta(S_5 + S_B) = \int d^5x\, e_4[\hat{D}_m(-4\alpha_1\psi_{n1}\sigma^{nm}\eta_1) + 6i\widetilde{M}\psi_{m1}\sigma^m\overline{\eta}_1 + \text{H.c.}]\delta(z), \tag{2.22}$$

where

$$\widetilde{M} = \lambda_1(1+3\alpha_1\alpha_1^*\varepsilon^2) + \lambda[\alpha_1 q_{12}^* + \alpha_1^* q_{12} + (3\alpha_1\alpha_1^*\varepsilon^2 - 1)q_3]. \tag{2.23}$$

The derivative term in Eq. (2.22) integrates to zero because the hatted derivative, defined in Eq. (A13), reduces to the covariant derivative on Σ. The other term vanishes, $\widetilde{M}\delta(z) = 0$, because of Eq. (2.17) and the fact that $\varepsilon^2\delta(z) = \frac{1}{3}\delta(z)$. Therefore the full bulk-plus-brane action is supersymmetric, without any further conditions.

Equation (2.17) defines the brane tension λ_1 in terms of α_1,

$$\lambda_1 = -\frac{\alpha_1 q_{12}^* + \alpha_1^* q_{12} + (\alpha_1\alpha_1^* - 1)q_3}{1+\alpha_1\alpha_1^*}\lambda. \tag{2.24}$$

Using the (complex) Cauchy-Buniakovsky-Schwarz inequality, $|\vec{a}\cdot\vec{b}|^2 \leqslant |\vec{a}|^2|\vec{b}|^2$, we find

$$\left(\frac{\lambda_1}{\lambda}\right)^2 \leqslant \vec{q}^2 = 1. \qquad (2.25)$$

This equation places an upper limit on the absolute value of the brane tension.

C. Two branes

This construction can be readily generalized to include a second brane. We now take the x^5 direction to have the topology of a circle S^1, and use the following parametrization for the fifth dimension:

$$S^1 = [-z_2, -z_1] \cup [z_1, z_2],$$
$$\varepsilon(z) = \begin{cases} +1, & z \in S^1_+ \equiv \quad (z_1, z_2), \\ -1, & z \in S^1_- \equiv \quad (-z_2, -z_1), \end{cases} \qquad (2.26)$$

where we identify $-z_1 \equiv z_1$ and $-z_2 \equiv z_2$. With these definitions, $\varepsilon'(z)$ changes to

$$\varepsilon'(z) = 2[\delta(z - z_1) - \delta(z - z_2)] \equiv 2[\delta_1(z) - \delta_2(z)]. \qquad (2.27)$$

The parity operation identifies z with $-z$. It gives rise to two fixed points, located at z_1 and z_2. As before, we place 3-branes at the fixed points. We take all fields and parameters to have the parity assignments (2.4), so their values in $z \in S^1_-$ are completely determined by those in $z \in S^1_+$. We work with fields redefined following Eq. (2.5).

We construct the supersymmetric bulk-plus-brane action by:

(i) introducing independent brane actions at the fixed points Σ_1 and Σ_2,

$$\begin{aligned} S_B &= \int_{\Sigma_1} d^4x e_4[-3\lambda_1 - 2\alpha_1 \psi_{m1}\sigma^{mn}\psi_{n1} + \text{H.c.}] \\ &\quad - \int_{\Sigma_2} d^4x e_4[-3\lambda_2 - 2\alpha_2 \psi_{m1}\sigma^{mn}\psi_{n1} + \text{H.c.}] \\ &= \int d^5x e_4\{-3[\lambda_1\delta_1(z) - \lambda_2\delta_2(z)] \\ &\quad - 2[\alpha_1\delta_1(z) - \alpha_2\delta_2(z)]\psi_{m1}\sigma^{mn}\psi_{n1} + \text{H.c.}\}; \end{aligned} \qquad (2.28)$$

(ii) modifying the supersymmetry transformations,

$$\delta\psi_{52} = \delta\psi_{52}|_{\text{old}} - 4[\alpha_1\delta_1(z) - \alpha_2\delta_2(z)]\eta_1; \qquad (2.29)$$

(iii) and imposing the following boundary conditions[3] on $\Sigma_{1,2}$:

$$\begin{aligned} & e_m^{\hat{5}} = e_5^a = B_m = 0, \quad \omega_{ma\hat{5}} = \varepsilon(z)\lambda_{1,2}e_{ma}, \\ & \eta_2 = \alpha_{1,2}\eta_1, \quad \psi_{m2} = \alpha_{1,2}\psi_{m1}, \\ & \psi_{51} = -\alpha^*_{1,2}\psi_{52}, \end{aligned} \qquad (2.30)$$

where $\alpha_{1,2} \in \mathbb{C}$ and $\lambda_{1,2} \in \mathbb{R}$ are constants, related as follows:[4]

$$\lambda_{1,2} = -\frac{\alpha_{1,2}q^*_{12} + \alpha^*_{1,2}q_{12} + (\alpha_{1,2}\alpha^*_{1,2} - 1)q_3}{1 + \alpha_{1,2}\alpha^*_{1,2}}\lambda. \qquad (2.31)$$

The brane tensions $T_1 = 6\lambda_1$ and $T_2 = -6\lambda_2$ are bounded by the inequality

$$|\lambda_{1,2}| \leqslant \lambda. \qquad (2.32)$$

In the next section we will see that the bulk-plus-brane system has a consistent dimensional reduction down to four dimensions. The resulting effective theory is $N=1$, $D=4$ (on-shell) supergravity with zero or negative cosmological constant.

III. EFFECTIVE ACTION

In this section we derive the effective action for the supergravity zero modes e_m^a and ψ_m. For simplicity, we ignore the radion multiplet and set the radion field at its expectation value. The zero modes for $e_m^{\hat{5}}$, e_5^a and B_m vanish because of the boundary condition (2.7).

For the following, we restore the gravitational coupling k_5 by rescaling the action and fields as

$$S \to k_5^2 S, \quad B_M \to k_5 B_M, \quad \psi_M \to k_5 \psi_M. \qquad (3.1)$$

A. Bosonic reduction

We first carry out the dimensional reduction for the bosonic part of the action. We take our ansatz to be

$$\begin{aligned} & e_m^a(x,z) = a(z)\hat{e}_m^a(x), \quad e_m^{\hat{5}} = e_5^a = 0, \\ & e_5^{\hat{5}} = 1, \quad B_m = B_5 = 0. \end{aligned} \qquad (3.2)$$

With this ansatz, the five-dimensional interval is

$$ds^2 = g_{MN}dx^M dx^N = a^2(z)\hat{g}_{mn}(x)dx^m dx^n + dz^2, \qquad (3.3)$$

and the connection coefficients are

$$\omega_{mab} = \hat{\omega}_{mab}, \quad \omega_{ma\hat{5}} = -a'(z)\hat{e}_{ma}, \quad \omega_{5ab} = \omega_{5a5} = 0, \qquad (3.4)$$

where $\hat{\omega}_{mab}$ is the four-dimensional connection for $\hat{e}_m^a$.

[3]The boundary conditions are determined by the parity assignments and the jump conditions that follow from the brane action. We could have worked on the interval (instead of the covering space with the orbifold identifications), in which case the boundary conditions guarantee that the supersymmetry variation of the bulk action (with arbitrary $\vec{q}$) has no residual boundary term.

[4]The results of [2] and [3–5] follow from ours if one sets $\vec{q} = (-1,0,0)$, $\alpha_1 = \alpha_2 = 1$ and $\vec{q} = (0,0,1)$, $\alpha_1 = \alpha_2 = 0$, respectively. In each case $\lambda_1 = \lambda_2 = \lambda$, which corresponds to the original Randall-Sundrum scenario with opposite-tension branes. The case with $|\lambda_1| = |\lambda_2| \leqslant \lambda$ and $\alpha_1 = \alpha_2 = 0$ was discussed in Ref. [9].

Since $B_M=0$, the bosonic part of the bulk-plus-brane action is

$$k_5^2 S_B = \int d^5x e_5\left(-\frac{1}{2}R-\Lambda_5\right) - \int_{\Sigma_1} d^4x e_4 T_1 - \int_{\Sigma_2} d^4x e_4 T_2, \tag{3.5}$$

where $\Lambda_5=-6\lambda^2$, $T_1=6\lambda_1$, and $T_2=-6\lambda_2$. The five-dimensional Einstein equations are

$$G_{mn}=[\Lambda_5+T_1\delta_1(z)+T_2\delta_2(z)]g_{mn},$$
$$G_{m5}=0, \quad G_{55}=\Lambda_5, \tag{3.6}$$

where $G_{MN}=R_{MN}-\frac{1}{2}Rg_{MN}$ is the five-dimensional Einstein tensor.

The $G_{m5}=0$ equation is trivially satisfied for our ansatz. The other two equations reduce to

$$\hat{G}_{mn}=\{3(aa''+a'^2)-6\lambda^2a^2 + 6[\lambda_1\delta_1(z)-\lambda_2\delta_2(z)]a^2\}\hat{g}_{mn} \tag{3.7}$$

$$\hat{R}=-12(a'^2-\lambda^2a^2), \tag{3.8}$$

where $\hat{G}_{mn}$ and $\hat{R}$ are the four-dimensional Einstein tensor and scalar curvature for the metric $\hat{g}_{mn}$.

Using separation of variables, we split Eq. (3.7) into

$$\hat{G}_{mn}=\Lambda_4\hat{g}_{mn}, \tag{3.9}$$

and

$$3(aa''+a'^2)-6\lambda^2a^2+6[\lambda_1\delta_1(z)-\lambda_2\delta_2(z)]a^2=\Lambda_4, \tag{3.10}$$

which are equations for $\hat{g}_{mn}(x)$ and $a(z)$, respectively. The separation constant Λ_4 is the cosmological constant in four dimensions, according to Eq. (3.9). We will soon find that supersymmetry requires $\Lambda_4\leqslant 0$. For this case we write

$$\Lambda_4=-3\lambda^2K^2. \tag{3.11}$$

[For the bosonic reduction alone, the case with a positive cosmological constant is obtained by replacing $K^2\to -K^2$ here and in Eq. (3.14) below.]

The delta functions in Eq. (3.10) must be canceled by corresponding singularities of $a''(z)$. Since $a(z)$ is an even function on $S^1/\mathbb{Z}_2$, we write

$$a(z)=a_0(y), \quad y=\varepsilon(z)\lambda z=\lambda|z|, \tag{3.12}$$

where $a_0(y)$ is a smooth function on $\mathbb{R}$. The derivatives of $a_0(y)$ are well-defined, and we have

$$a''(z)=\lambda^2a_0''(y)+2\lambda[\delta_1(z)-\delta_2(z)]a_0'(y). \tag{3.13}$$

With these redefinitions, Eqs. (3.7) and (3.8) are simply

$$a_0''=a_0, \quad a_0'^2=a_0^2-K^2, \tag{3.14}$$

with boundary conditions

$$\lambda_{1,2}=-\lambda\frac{a_0'}{a_0}(y_{1,2}). \tag{3.15}$$

The latter follow from the boundary conditions for the connection $\omega_{ma\hat{5}}$.

We now proceed to find the effective action, without explicitly solving for the warp factor a_0. Indeed, using Eqs. (3.14) and (3.15), together with

$$R=a^{-2}(\hat{R}+8aa''+12a'^2), \tag{3.16}$$

and

$$\oint dz a_0^3a_0'[\delta_1(z)-\delta_2(z)]=-\frac{\lambda}{2}\oint dz(a_0^3a_0')', \tag{3.17}$$

we cast Eq. (3.5) in the form

$$S_B=\frac{1}{k_5^2}\oint dz a_0^2\int d^4x\hat{e}_4\left(-\frac{1}{2}\hat{R}-\Lambda_4\right), \tag{3.18}$$

where $\hat{e}_4=\det(\hat{e}_m^a)=\sqrt{-\det(\hat{g}_{mn})}$. If we define the effective four-dimensional gravitational coupling to be

$$\frac{1}{k_4^2}=\frac{1}{k_5^2}\oint dz a_0^2, \tag{3.19}$$

we recover the action for four-dimensional gravity with a cosmological constant Λ_4,

$$S_B=\frac{1}{k_4^2}\int d^4x\hat{e}_4\left(-\frac{1}{2}\hat{R}-\Lambda_4\right). \tag{3.20}$$

Equations (3.9) follow from this action, so the dimensional reduction is consistent.

It is easy to solve Eqs. (3.14) to find the explicit form for the bosonic warp factor [10]:

(a) $\Lambda_4=-3\lambda^2K^2, \quad AdS_5\to AdS_4, \quad |\lambda_{1,2}|<\lambda$

$a_0(y)=K\cosh(y-y_0)$

(b) $\Lambda_4=0, \quad AdS_5\to Mink_4, \quad \lambda_1=\lambda_2=\mp\lambda$

$a_0(y)=\exp(\pm(y-y_0))$

(c) $\Lambda_4=+3\lambda^2K^2, \quad AdS_5\to dS_4, \quad |\lambda_{1,2}|>\lambda$

$a_0(y)=K\sinh(y-y_0)$. (3.21)

(We use $dS/AdS/Mink$ to denote a theory with a positive/negative/zero cosmological constant.) The restrictions on $\lambda_{1,2}$ follow from boundary conditions (3.15) and the fact that

$|\tanh(y)|<1$. The case $\Lambda_4=0$ corresponds to the usual Randall-Sundrum scenario with two opposite-tension branes.

In the previous section we found that local supersymmetry places a restriction (2.32) on the brane tensions: $|\lambda_{1,2}| \leqslant\lambda$. This implies, according to Eq. (3.21), that local supersymmetry restricts the effective four-dimensional theory to be AdS_4 or $Mink_4$. Note that for $Mink_4$, the parameter y_0 in Eq. (3.21) is arbitrary and interbrane distance is not fixed. For AdS_4 the boundary conditions (3.15) imply $y_0 = y_1 + \operatorname{arctanh}(\lambda_1/\lambda)$ and fix the proper distance Δz in terms of λ_1, λ_2 and λ:

$$\lambda\Delta z = \operatorname{arctanh}\left(\frac{\lambda_1}{\lambda}\right) - \operatorname{arctanh}\left(\frac{\lambda_2}{\lambda}\right) = \frac{1}{2}\ln\left[\frac{(\lambda+\lambda_1)(\lambda-\lambda_2)}{(\lambda+\lambda_2)(\lambda-\lambda_1)}\right]. \tag{3.22}$$

The effective cosmological constant Λ_4 is determined once we normalize the bosonic warp factor. It is natural to require that $a_0(y)$ is unity at the location of an observer (so that s/he uses the same time and distance scales for both the five-dimensional and effective four-dimensional theory). When effective theory is AdS_4, we use $\cosh(y)\geqslant 1$ to find that $0<K\leqslant 1$ and

$$-3\lambda^2 \leqslant \Lambda_4 < 0. \tag{3.23}$$

B. Fermionic reduction

The fermionic part of the four-dimensional effective action is fixed by supersymmetry. The five-dimensional gravitini must reduce to a four-dimensional gravitino $\psi_m(x)$, while the five-dimensional supersymmetry parameters must reduce to a four-dimensional spinor $\eta(x)$. These considerations motivate the following ansatz for the fermionic fields:

$$\eta_1(x,z) = \beta_1(y)\,\eta(x), \qquad \eta_2(x,z) = \beta_2(y)\,\eta(x),$$
$$\psi_{m1}(x,z) = \gamma\beta_1(y)\,\psi_m(x), \quad \psi_{51}(x,z)=0,$$
$$\psi_{m2}(x,z) = \gamma\beta_2(y)\,\psi_m(x), \quad \psi_{52}(x,z)=0, \tag{3.24}$$

where the complex warp factors are functions of $y=\lambda|z|$, just like a_0 is a function of y in Eq. (3.12).

Supersymmetry imposes the following consistency conditions on this ansatz:

$$\beta_1\delta\psi_{m2} = \beta_2\delta\psi_{m1}, \quad \delta\psi_{51}=0, \quad \delta\psi_{52}=0. \tag{3.25}$$

The first condition requires

$$(\beta_1\beta_1^* + \beta_2\beta_2^*)\frac{a_0'}{a_0} = q_{12}\beta_1\beta_2^* + q_{12}^*\beta_1^*\beta_2 + q_3(\beta_2\beta_2^* - \beta_1\beta_1^*). \tag{3.26}$$

This equation, restricted to the fixed points, together with boundary conditions

$$\alpha_{1,2} = \frac{\beta_2}{\beta_1}(y_{1,2}) \tag{3.27}$$

and Eq. (3.15), implies the relation (2.31) between $\lambda_{1,2}$ and $\alpha_{1,2}$. The other two conditions, $\delta\psi_{51}=0$ and $\delta\psi_{52}=0$, give rise to the following equations:

$$2\beta_1' = q_{12}^*\beta_2 - q_3\beta_1, \quad 2\beta_2' = q_{12}\beta_1 + q_3\beta_2. \tag{3.28}$$

They permit the right-hand side of Eq. (3.26) to be written as $(\beta_1\beta_1^* + \beta_2\beta_2^*)'$, which implies that the bosonic and fermionic warp factors obey

$$a_0 = \beta_1\beta_1^* + \beta_2\beta_2^*, \tag{3.29}$$

up to a multiplicative constant that we set equal to unity.

These conditions are sufficient to find the supersymmetry transformations of the four-dimensional fields. Using the five-dimensional transformation δe_m^a, together with the combination $\beta_1^*\delta\psi_{m1} + \beta_2^*\delta\psi_{m2}$, we find

$$\delta\hat{e}_m^a = ik_5\gamma^*\eta\sigma^a\overline{\psi_m} + \text{H.c.}$$
$$k_5\gamma\delta\psi_m = 2\hat{D}_m\eta + i\lambda g\hat{\sigma}_m\overline{\eta}, \tag{3.30}$$

where

$$g^* = q_{12}\beta_1^2 - q_{12}^*\beta_2^2 + 2q_3\beta_1\beta_2. \tag{3.31}$$

Equations (3.28) and (3.29) assure us that g is a constant, and relate it to K as follows:

$$gg^* = a_0^2 - a_0'^2 = K^2. \tag{3.32}$$

The four-dimensional supersymmetry transformations (3.30) take their usual form if we set

$$\gamma = \frac{k_4}{k_5}. \tag{3.33}$$

To find the effective action, we first note that the $\psi_{m1}\sigma^{mn}\psi_{n2}\delta_{1,2}(z)$ terms from the bulk action exactly cancel the $\alpha_{1,2}\psi_{m1}\sigma^{mn}\psi_{n1}\delta_{1,2}(z)$ terms from the branes. Therefore the fermionic part of the bulk-plus-brane action reduces to

$$S_F=\int d^5xe_4\left\{\frac{1}{2}\epsilon^{mpnk}(\overline{\psi}_{m1}\overline{\sigma}_pD_n\psi_{k1}+\overline{\psi}_{m2}\overline{\sigma}_pD_n\psi_{k2})+\varepsilon(z)(\psi_{m1}\sigma^{mn}\partial_5\psi_{n2}-\psi_{m2}\sigma^{mn}\partial_5\psi_{n1})\right.$$
$$\left.-\frac{3\lambda}{2}(q_{12}\psi_{m1}\sigma^{mn}\psi_{n1}-q_{12}^*\psi_{m2}\sigma^{mn}\psi_{n2}+2q_3\psi_{m1}\sigma^{mn}\psi_{n2})+\text{H.c.}\right\}. \tag{3.34}$$

Using Eqs. (3.28), (3.29) and (3.31), as well as

$$\sigma_m=a_0\hat{\sigma}_m,\quad \sigma^{mn}=a_0^{-2}\hat{\sigma}^{mn},\quad \epsilon^{mpnk}=a_0^{-4}\hat{\epsilon}^{mpnk},\quad e_4=a_0^4\hat{e}_4, \tag{3.35}$$

we transform this action to

$$S_F=\gamma^2\oint dza_0^2\int d^4x\hat{e}_4\left[\frac{1}{2}\hat{\epsilon}^{mpnk}\overline{\psi}_m\hat{\overline{\sigma}}_p\hat{D}_n\psi_k-\lambda g^*\psi_m\hat{\sigma}^{mn}\psi_n+\text{H.c.}\right]. \tag{3.36}$$

The normalization conditions (3.19) and (3.33) ensure that $\gamma^2\oint dza_0^2=1$. Together with the bosonic part, the total effective action is therefore

$$S_4=\int d^4x\hat{e}_4\left\{\frac{1}{k_4^2}\left(-\frac{1}{2}\hat{R}+3\lambda^2gg^*\right)+\left[\frac{1}{2}\hat{\epsilon}^{mpnk}\overline{\psi}_m\hat{\overline{\sigma}}_p\hat{D}_n\psi_k-\lambda g^*\psi_m\hat{\sigma}^{mn}\psi_n+\text{H.c.}\right]\right\}, \tag{3.37}$$

which is the correct action for locally $N=1$ supersymmetric theory in four dimensions. This completes the dimensional reduction.

If we restrict $\hat{g}_{mn}(x)$ to be the metric for the maximally symmetric anti–de Sitter (or Minkowski) background, the local supersymmetry breaks to $N=1$ global supersymmetry. The unbroken supersymmetry is described by the five-dimensional spinors $\eta_{1,2}=\beta_{1,2}(y)\eta(x)$, where $\eta(x)$ is now fixed to be the four-dimensional Killing spinor in the $\hat{g}_{mn}(x)$ background [11].

We also present here explicit solutions for the fermionic warp factors. Equations (3.28) are straightforward to solve. We first note that $4\beta''_{1,2}=\beta_{1,2}$ and, using the boundary conditions (3.27), we obtain the following expressions:

$$\beta_1(y)=b_0\cosh\tfrac{1}{2}(y-y_1)+(q_{12}^*\alpha_1-q_3)b_0\sinh\tfrac{1}{2}(y-y_1) \tag{3.38}$$

$$\beta_2(y)=\alpha_1b_0\cosh\tfrac{1}{2}(y-y_1)+(q_{12}+\alpha_1q_3)b_0\sinh\tfrac{1}{2}(y-y_1). \tag{3.39}$$

The overall constant b_0 is fixed (up to a phase) by the normalization of the bosonic warp factor. Substituting these expressions into Eq. (3.31), we find

$$g^*=b_0^2(q_{12}-q_{12}^*\alpha_1^2+2q_3\alpha_1). \tag{3.40}$$

We distinguish two cases. If α_1 is a root of

$$q_{12}-q_{12}^*\alpha_1^2+2q_3\alpha_1=0, \tag{3.41}$$

the effective theory is $Mink_4$. This equation implies $q_{12}^*\alpha_1-q_3=\pm1$ and $q_{12}+q_3\alpha_1=\pm\alpha_1$, which permits us to write

$$\beta_1(y)=b_0\exp(\pm\tfrac{1}{2}y),\quad \beta_2(y)=\alpha_1\beta_1(y),$$
$$a_0(y)=\frac{2b_0b_0^*}{1\mp q_3}\exp(\pm y). \tag{3.42}$$

The boundary conditions (3.15) and (3.27) require $\alpha_2=\alpha_1$ and $\lambda_2=\lambda_1=\mp\lambda$, independent of the interbrane distance.

For any complex α_1 which is not a solution of Eq. (3.41), the effective theory is AdS_4. The value of λ_1 is determined by α_1,

$$\lambda_1=-\frac{\alpha_1q_{12}^*+\alpha_1^*q_{12}+(\alpha_1\alpha_1^*-1)q_3}{1+\alpha_1\alpha_1^*}\lambda, \tag{3.43}$$

so $|\lambda_1|<\lambda$. We introduce the real variable $\hat{y}_1=\text{arctanh}(\lambda_1/\lambda)$ and, using Eqs. (3.29) and (3.38), cast the bosonic warp factor in the following form:

$$a_0(y)=K\cosh[y-(y_1+\hat{y}_1)], \tag{3.44}$$

where

$$K=|g|=\frac{b_0b_0^*(1+\alpha_1\alpha_1^*)}{\cosh\hat{y}_1}. \tag{3.45}$$

For a given separation of the branes, $\Delta y=y_2-y_1$, the boundary conditions (3.15) and (3.27) determine the values of α_2 and λ_2,

$$\alpha_2=\frac{\alpha_1+(q_{12}+\alpha_1q_3)\tanh(\tfrac{1}{2}\Delta y)}{1+(q_{12}^*\alpha_1-q_3)\tanh(\tfrac{1}{2}\Delta y)},$$
$$\lambda_2=\lambda\frac{\lambda_1-\lambda\tanh\Delta y}{\lambda-\lambda_1\tanh\Delta y}. \tag{3.46}$$

It is not hard to check that α_2 and λ_2 are related by Eq. (2.31) and furthermore, that α_2 cannot be a solution of Eq. (3.41). Alternatively, for a given α_1 and α_2, the interbrane separation is

$$\Delta y = 2\,\mathrm{arctanh}\left(\frac{\alpha_2-\alpha_1}{q_{12}-q_{12}^{*}\alpha_1\alpha_2+q_3(\alpha_1+\alpha_2)}\right). \tag{3.47}$$

This equation is equivalent to Eq. (3.22), since $\lambda_{1,2}$ are given by Eq. (2.31).

The fact that the argument of arctanh must be real and of absolute value less than unity implies that α_2 cannot be chosen completely independently of α_1. For example, if $q_3=1$, this restricts α_1 and α_2 to have the same complex phase,

$$\alpha_1 = r_1 e^{i\theta}, \quad \alpha_2 = r_2 e^{i\theta}. \tag{3.48}$$

For given $\lambda_{1,2}$, the absolute values of $\alpha_{1,2}$ are determined as

$$r_{1,2} = \cosh \hat{y}_{1,2} - \sinh \hat{y}_{1,2}, \quad \hat{y}_{1,2} = \mathrm{arctanh}\left(\frac{\lambda_{1,2}}{\lambda}\right). \tag{3.49}$$

For $q_3 \neq 1$, one can first rotate to $\vec{q}\,' = (0,0,1)$, as explained in Appendix B, then use the above result and rotate back, arriving at

$$\alpha_1 = \frac{1-q_3+q_{12}r_1e^{i\theta}}{(1-q_3)r_1e^{i\theta}-q_{12}^{*}}, \quad \alpha_2 = \frac{1-q_3+q_{12}r_2e^{i\theta}}{(1-q_3)r_2e^{i\theta}-q_{12}^{*}}, \tag{3.50}$$

where $r_{1,2}$ are given by the same expressions, and θ is arbitrary. The apparent singularity at $q_3=1$ is a consequence of trying to cover the sphere $\vec{q}^{\,2}=1$ using a single coordinate patch.

IV. SUMMARY AND CONCLUSIONS

In this paper we presented a general bulk-plus-brane action for the supersymmetric Randall-Sundrum scenario. The bulk action is that of $N=2$ supergravity, compactified on a five-dimensional $S^1/\mathbb{Z}_2$ orbifold. The brane action contains supergravity fields induced from the bulk. The bulk gravitino mass depends on a vector $\vec{q}$, parametrizing a point on the sphere S^2.

We demonstrated that our bulk-plus-brane action has local $N=2$ supersymmetry, constrained by boundary conditions for the fields and supersymmetry parameters. The boundary conditions are those implied by consistency of the five-dimensional equations of motion and supersymmetry transformations. For the action we considered, the brane tensions $T_1=6\lambda_1$ and $T_2=-6\lambda_2$ respect an upper limit, expressed in terms of the bulk cosmological constant, $\Lambda_5=-6\lambda^2$, as $|\lambda_{1,2}|\leq\lambda$.

We also presented a consistent dimensional reduction for the bulk-plus-brane system. We derived the action and the supersymmetry transformations in a background-independent way, without explicitly solving for the warp factors. The effective action is that of minimal $N=1$ supergravity in four dimensions, with zero or negative cosmological constant. The effective cosmological constant is zero if and only if $\lambda_1=\lambda_2=\pm\lambda$, which corresponds to the original Randall-Sundrum scenario with two opposite-tension branes. Our results show, however, that the Randall-Sundrum fine-tuning is not a consequence of supersymmetry. For all other $|\lambda_{1,2}|<\lambda$, we obtain an $N=1$ supersymmetric theory with a negative cosmological constant, limited by

$$\frac{1}{2}\Lambda_5 \leq \Lambda_4 < 0. \tag{4.1}$$

When Λ_5 is nonzero, the gravitino mass localized on each brane is determined by the brane tension and the bulk cosmological constant. In contrast to flat space, supersymmetry cannot be broken spontaneously by changing the brane masses, as was done in [12]. Spontaneous supersymmetry breaking for warped geometries is discussed in [9,13,14].

Note added in proof. When $\Lambda_5\neq 0$, the $SU(2)$ automorphism symmetry of the bulk action is broken to a $U(1)$ R symmetry that depends on $\vec{q}$. The transformation $\Psi_{mi} \to \Psi'_{mi} = U_i^{\ j}\Psi_{mj}$ leaves the bulk action (2.1) invariant for $U=\exp[i(\vec{q}\cdot\vec{\sigma})\phi]$, where $\phi\in\mathbb{R}$. It is a symmetry of the full theory if it also preserves the boundary conditions (2.9). This is the case precisely when Eq. (3.41) is satisfied, that is, when effective theory has $\Lambda_4=0$. The $U(1)$ R symmetry of the five-dimensional theory gives rise to a $U(1)$ R symmetry of the effective theory, $\psi_m\to\psi'_m=\exp(\mp i\phi)\psi_m$. We thank A. Nelson for raising this question.

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APPENDIX A: CONVENTIONS

In this paper we adopt the following index conventions:

M,N,P,Q,K	coordinate space	$M=\{m,5\}$	$m=\{0,1,2,3\}$
A,B,C,D,E	tangent space	$A=\{a,\hat{5}\}$	$a=\{\hat{0},\hat{1},\hat{2},\hat{3}\}$
i,j	$SU(2)$	$i=\{1,2\}$.	

(A1)

We denote the determinant of an n-bein by e_n:

$$e_5=\det e_M^A, \quad e_4=\det e_m^a, \quad \hat{e}_4=\det \hat{e}_m^a. \tag{A2}$$

The fünfbein e_M^A (and the veirbein e_m^a) allow one to convert between the two types of indices:

$$\Gamma_M = e_M^A\Gamma_A, \quad g_{MN}=e_M^A e_N^B \eta_{AB},$$
$$\epsilon^{MNPQK} = e_A^M e_B^N e_C^P e_D^Q e_E^K \epsilon^{ABCDE}. \tag{A3}$$

The gamma matrices obey the following relations:

$$\{\Gamma_A,\Gamma_B\}=-2\eta_{AB},\quad \eta_{AB}=\mathrm{diag}(-++++)$$
$$\Gamma^{ABCDE}=-\epsilon^{ABCDE},\quad \epsilon^{\hat{0}\hat{1}\hat{2}\hat{3}\hat{5}}=+1,$$
$$\epsilon^{abcd\hat{5}}=\epsilon^{abcd} \tag{A4}$$

and

$$\Gamma^{ABCD}=\epsilon^{ABCDE}\Gamma_E,\quad \Gamma^{ABC}=\epsilon^{ABCDE}\Sigma_{DE},$$
$$\Gamma^{AB}=\tfrac{1}{2}[\Gamma^A,\Gamma^B]=2\Sigma^{AB}. \tag{A5}$$

The reduction to two-component notation [15] exploits the following representation for the gamma matrices:

$$\Gamma^a=\begin{pmatrix}0 & \sigma^a\\ \overline{\sigma}^a & 0\end{pmatrix},$$
$$\Gamma^{\hat{5}}=\begin{pmatrix}-i & 0\\ 0 & i\end{pmatrix}\Rightarrow\Sigma^{ab}=\begin{pmatrix}\sigma^{ab} & 0\\ 0 & \overline{\sigma}^{ab}\end{pmatrix},$$
$$\Sigma^{a\hat{5}}=\frac{i}{2}\begin{pmatrix}0 & \sigma^a\\ -\overline{\sigma}^a & 0\end{pmatrix}. \tag{A6}$$

The charge conjugation matrix is taken to be

$$C=\begin{pmatrix}i\sigma_2 & 0\\ 0 & i\sigma_2\end{pmatrix}=\begin{pmatrix}-\epsilon_{\alpha\beta} & 0\\ 0 & \epsilon^{\dot{\alpha}\dot{\beta}}\end{pmatrix}. \tag{A7}$$

With this representation, a four-component Dirac spinor, its Dirac conjugate and its Majorana conjugate are

$$\Psi=\begin{pmatrix}\psi_{1\alpha}\\ \overline{\psi}_2^{\dot{\alpha}}\end{pmatrix},\quad \overline{\Psi}=\Psi^\dagger\Gamma_{\hat{0}}=(\psi_2^\alpha,\overline{\psi}_{1\dot{\alpha}}),$$
$$\widetilde{\Psi}=\Psi^TC=(-\psi_1^\alpha,\overline{\psi}_{2\dot{\alpha}}). \tag{A8}$$

A symplectic Majorana spinor obeys the following condition:

$$\widetilde{\Psi}^i=\overline{\Psi}_i. \tag{A9}$$

We take

$$\Psi_1=-\Psi^2=\begin{pmatrix}\psi_1\\ \overline{\psi}_2\end{pmatrix},\quad \Psi_2=\Psi^1=\begin{pmatrix}-\psi_2\\ \overline{\psi}_1\end{pmatrix}. \tag{A10}$$

The covariant derivative and its commutator on a Dirac spinor are given by

$$D_M\Psi=\partial_M\Psi+\tfrac{1}{2}\omega_{MAB}\Sigma^{AB}\Psi,$$
$$[D_M,D_N]\Psi=\tfrac{1}{2}R_{MNAB}\Sigma^{AB}\Psi. \tag{A11}$$

The connection coefficients, curvature tensor and scalar curvature are defined as follows:

$$\omega_{MAB}=\tfrac{1}{2}e_A^Ne_B^K(e_{MC}\partial_{[N}e_{K]}^C-e_{NC}\partial_{[K}e_{M]}^C$$
$$-e_{KC}\partial_{[M}e_{N]}^C)$$
$$R_{MNAB}=\partial_M\omega_{NAB}-\partial_N\omega_{MAB}$$
$$+\omega_{NA}{}^C\omega_{MCB}-\omega_{MA}{}^C\omega_{NCB}$$
$$R=e^{MA}R_{MA}=e^{MA}e^{NB}R_{MNAB}. \tag{A12}$$

We introduce a hatted covariant derivative by splitting D_M as

$$D_M\Psi=\hat{D}_M\Psi+\omega_{Ma\hat{5}}\Sigma^{a\hat{5}}\Psi$$
$$\Rightarrow\begin{cases}D_M\psi_1=\hat{D}_M\psi_1+\dfrac{i}{2}\omega_{Ma\hat{5}}\sigma^a\overline{\psi}_2,\\ D_M\psi_2=\hat{D}_M\psi_2-\dfrac{i}{2}\omega_{Ma\hat{5}}\sigma^a\overline{\psi}_1,\\ \hat{D}_M\psi=\partial_M\psi+\dfrac{1}{2}\omega_{Mab}\sigma^{ab}\psi.\end{cases} \tag{A13}$$

When $e_m^{\hat{5}}=e_5^a=0$, $\hat{D}_m$ becomes the covariant derivative for e_m^a. When, in addition, $e_m^a=a(z)\hat{e}_m^a(x)$, $\hat{D}_m$ is also the covariant derivative for $\hat{e}_m^a(x)$.

APPENDIX B: THE BULK SUPERGRAVITY ACTION

The action of pure $N=2$, $D=5$ supergravity without a cosmological constant is invariant under the $SU(2)$ rotations $\Psi'_{Mi}=U_i{}^j\Psi_{Mj}$. The rotation $U\in SU(2)$ can be written in terms of the Pauli matrices,

$$U^\dagger U=1,\quad \det(U)=1\quad\Rightarrow\quad U_i{}^j=u_0\sigma_0+i\vec{u}\cdot\vec{\sigma},$$
$$u_0,u_i\in\mathbb{R},\quad u_0^2+\vec{u}^2=1. \tag{B1}$$

A cosmological constant is introduced by gauging a $U(1)$ subgroup of the $SU(2)$. The gauge coupling breaks the symmetry and changes the covariant derivative $D_M\Psi_{Ni}$ into $D_M\Psi_{Ni}-\sqrt{\tfrac{3}{2}}\lambda B_MQ_i{}^j\Psi_{Nj}$, where

$$Q^\dagger=-Q,\quad \mathrm{Tr}(Q)=0\quad\Rightarrow\quad Q_i{}^j=i\vec{q}\cdot\vec{\sigma}=i\begin{pmatrix}q_3 & q_1-iq_2\\ q_1+iq_2 & -q_3\end{pmatrix},\quad q_i\in\mathbb{R}. \tag{B2}$$

The matrix for the $SU(2)$ rotation on the two-component spinors, $\psi'_i=\widetilde{U}_i{}^j\psi_j$, is given by

$$\widetilde{U}=u_0\sigma_0+i(-u_1\sigma_1-u_2\sigma_2+u_3\sigma_3)=\sigma_3U\sigma_3. \tag{B3}$$

Any such rotation can be compensated by changing Q,

$$Q' = UQU^\dagger = (u_0 + i\vec{u}\cdot\vec{\sigma}) i\vec{q}\cdot\vec{\sigma}(u_0 - i\vec{u}\cdot\vec{\sigma}) = i\vec{q}\,'\cdot\vec{\sigma}$$

$$\vec{q}\,' = \vec{q} + 2\vec{u}\times(\vec{u}\times\vec{q}) - 2u_0(\vec{u}\times\vec{q}). \tag{B4}$$

With these conventions, the action of gauged supergravity is[5]

$$S_5 = \int d^5x e_5 \Big\{ -\frac{1}{2}R + 6\lambda^2\vec{q}^2 + \frac{i}{2}\widetilde{\Psi}^i_M \Gamma^{MNK} D_N \Psi_{Ki} + i\frac{3}{2}\lambda\widetilde{\Psi}^i_M \Sigma^{MN} Q_i{}^j \Psi_{Nj} - \frac{1}{4}F_{MN}F^{MN}$$
$$- i\frac{\sqrt{6}}{16}F_{MN}(2\widetilde{\Psi}^{Mi}\Psi^N_i + \widetilde{\Psi}^i_P \Gamma^{MNPQ}\Psi_{Qi}) - \frac{1}{6\sqrt{6}}\epsilon^{MNPQK}F_{MN}F_{PQ}B_K - i\frac{\sqrt{6}}{4}\lambda B_N \widetilde{\Psi}^i_M \Gamma^{MNK} Q_i{}^j \Psi_{Kj} \Big\}. \tag{B5}$$

For constant parameters, the Lagrangian is invariant (up to a total derivative) under the following supersymmetry transformations:

$$\delta e^A_M = i\widetilde{\mathcal{H}}^i \Gamma^A \Psi_{Mi}$$

$$\delta B_M = i\frac{\sqrt{6}}{2}\widetilde{\Psi}^i_M \mathcal{H}_i$$

$$\delta\Psi_{Mi} = 2\left(D_M\mathcal{H}_i - \frac{\sqrt{6}}{2}\lambda B_M Q_i{}^j \mathcal{H}_j\right) + \lambda\Gamma_M Q_i{}^j\mathcal{H}_j + \frac{1}{2\sqrt{6}}(\Gamma_{MNK} - 4g_{MK}\Gamma_N)F^{NK}\mathcal{H}_i. \tag{B6}$$

If the parameters are not constant, as when we change constant q_3 by a function $\varepsilon(z)q_3$, the variation of the action is

$$\delta S_5 = \int d^5x e_5 \{ D_M(\ldots)^M - [6i\widetilde{\Psi}^i_M \Sigma^{MN}\mathcal{H}_j + \sqrt{6}i\widetilde{\Psi}^i_M \Gamma^{MNK}\mathcal{H}_j B_K]\partial_N(\lambda Q_i{}^j)\}. \tag{B7}$$

The action and transformation laws can also be written in terms of two component spinors. We use the identities

$$i\widetilde{\Psi}^i \vec{\Gamma}\mathcal{H}_i = i\overline{\Psi}_1 \vec{\Gamma}\mathcal{H}_1 + \text{H.c.}$$

$$i\widetilde{\Psi}^i \vec{\Gamma} Q_i{}^j \mathcal{H}_j = -q_3 \overline{\Psi}_1 \vec{\Gamma}\mathcal{H}_1 - q_{12}\overline{\Psi}_2\vec{\Gamma}\mathcal{H}_1 + \text{H.c.}, \tag{B8}$$

where $q_{12} = q_1 + iq_2$, $\vec{\Gamma} = \Gamma^{A_1}\Gamma^{A_2}\ldots\Gamma^{A_n}$, and Ψ^i and $\mathcal{H}^i$ are arbitrary symplectic Majorana spinors. We carry out the redefinition (2.5) and set $e^{\hat{5}}_m = e^a_5 = B_m = 0$. The fermionic part of the bulk action is then

$$S_{5F} = \int d^5x e_5 e^{\hat{5}}_5 \Big\{ \frac{1}{2}\epsilon^{mpnk}(\varepsilon^2\overline{\psi}_{m2}\overline{\sigma}_p D_n\psi_{k2} + \overline{\psi}_{m1}\overline{\sigma}_p D_n\psi_{k1})e^{\hat{5}}_5 + (\psi_{52}\sigma^{mn}D_m\psi_{n1} - \varepsilon^2\psi_{51}\sigma^{mn}D_m\psi_{n2}) + (\varepsilon^2\psi_{m2}\sigma^{mn}D_n\psi_{51}$$
$$- \psi_{m1}\sigma^{mn}D_n\psi_{52}) - \frac{3\lambda}{2}\{\varepsilon^2 q_3[(\psi_{m2}\sigma^{mn}\psi_{n1} + \psi_{m1}\sigma^{mn}\psi_{n2})e^{\hat{5}}_5 + i(\psi_{m2}\sigma^m\overline{\psi}_{52} - \psi_{m1}\sigma^m\overline{\psi}_{51})] + q_{12}[(\psi_{m1}\sigma^{mn}\psi_{n1}$$
$$- \varepsilon^2\overline{\psi}_{m2}\overline{\sigma}^{mn}\overline{\psi}_{n2})e^{\hat{5}}_5 + i(\psi_{m1}\sigma^m\overline{\psi}_{52} + \varepsilon^2\overline{\psi}_{m2}\overline{\sigma}^m\psi_{51})]\} - \frac{\sqrt{6}}{4}ie^{\hat{5}}_5 F^{m5}(\varepsilon^2\psi_{m2}\psi_{51} - \psi_{m1}\psi_{52}) + \frac{\sqrt{6}}{8}iF_{m5}\epsilon^{mnpq}(\varepsilon^2\psi_{p2}\sigma_n\overline{\psi}_{q2}$$
$$+ \psi_{p1}\sigma_n\overline{\psi}_{q1}) + \frac{\sqrt{6}}{2}i\lambda B_5[\varepsilon^2 q_3(\psi_{m2}\sigma^{mn}\psi_{n1} + \psi_{m1}\sigma^{mn}\psi_{n2}) + q_{12}(\psi_{m1}\sigma^{mn}\psi_{n1} + \varepsilon^2\overline{\psi}_{m2}\overline{\sigma}^{mn}\overline{\psi}_{n2})] - \varepsilon(\psi_{m2}\sigma^{mn}D_5\psi_{n1}$$
$$- \psi_{m1}\sigma^{mn}D_5\psi_{n2}) + 2\psi_{m1}\sigma^{mn}\psi_{n2}\delta(z) + \text{H.c.}\Big\}, \tag{B9}$$

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[5]We assume $\vec{q}^2 = 1$. This makes the cosmological constant $\Lambda_5 = -6\lambda^2\vec{q}^2 = 3\lambda^2\mathrm{Tr}(Q^2)$ independent of $\vec{q}$. Our definition of $Q_i{}^j$ follows Ref. [5].

and the supersymmetry transformations are

$$\delta e_m^a = i(\varepsilon^2 \eta_2 \sigma^a \overline{\psi}_{m2} + \eta_1 \sigma^a \overline{\psi}_{m1}) + \text{H.c.}$$

$$\delta e_5^a = i(\eta_2 \sigma^a \overline{\psi}_{52} + \eta_1 \sigma^a \overline{\psi}_{51}) + \text{H.c.}$$

$$\delta e_m^{\hat{5}} = \eta_2 \psi_{m1} - \eta_1 \psi_{m2} + \text{H.c.}$$

$$\delta e_5^{\hat{5}} = \varepsilon^2 \eta_2 \psi_{51} - \eta_1 \psi_{52} + \text{H.c.}$$

$$\delta B_m = i\frac{\sqrt{6}}{2}(\psi_{m2}\eta_1 - \psi_{m1}\eta_2) + \text{H.c.}$$

$$\delta B_5 = i\frac{\sqrt{6}}{2}(\psi_{52}\eta_1 - \varepsilon^2 \psi_{51}\eta_2) + \text{H.c.}$$

$$\delta\psi_{m1} = 2D_m\eta_1 + i\lambda\sigma_m(\varepsilon^2 q_3 \overline{\eta}_2 + q_{12}^* \overline{\eta}_1) - \frac{2}{\sqrt{6}} i e_5^{\hat{5}} F^{n5}(\sigma_{mn} + g_{mn})\eta_1$$

$$\delta\psi_{m2} = 2D_m\eta_2 + i\lambda\sigma_m(q_3 \overline{\eta}_1 - q_{12} \overline{\eta}_2) - \frac{2}{\sqrt{6}} i e_5^{\hat{5}} F^{n5}(\sigma_{mn} + g_{mn})\eta_2$$

$$\delta\psi_{51} = 2\varepsilon^{-1} D_5\eta_1 + \lambda(e_5^{\hat{5}} - \sqrt{6} i B_5)(q_3\eta_1 - q_{12}^*\eta_2) - \frac{2}{\sqrt{6}} F_{m5}\sigma^m \overline{\eta}_2$$

$$\delta\psi_{52} = 2\varepsilon D_5\eta_2 - \lambda(e_5^{\hat{5}} - \sqrt{6} i B_5)(\varepsilon^2 q_3\eta_2 + q_{12}\eta_1) + \frac{2}{\sqrt{6}} F_{m5}\sigma^m \overline{\eta}_1 + 4\eta_2\delta(z), \tag{B10}$$

where

$$D_m\eta_1 = \hat{D}_m\eta_1 + \frac{i}{2}\varepsilon\,\omega_{ma\hat{5}}\sigma^a\overline{\eta}_2, \quad D_m\eta_2 = \hat{D}_m\eta_2 - \frac{i}{2}\varepsilon^{-1}\omega_{ma\hat{5}}\sigma^a\overline{\eta}_1, \tag{B11}$$

and similarly for other covariant derivatives, according to Eq. (A13). [The corresponding expressions *before* the redefinition (2.5) are obtained by setting $\varepsilon = 1$ and dropping the $\delta(z)$ terms.]

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